

# KAON PHOTOPRODUCTION OF THE DEUTERON AND THE $P$ -MATRIX APPROACH TO THE $YN$ FINAL STATE INTERACTION

B.O.Kerbikov

*Institute of Theoretical and Experimental Physics, 117218, Moscow, Russia*

Strangeness photoproduction of the deuteron is investigated theoretically making use of the covariant reaction formalism and the  $P$ -matrix approach the final state hyperon-nucleon interaction. Remarkably simple analytical expression for the amplitude is obtained. Pronounced effects due to final state interaction are predicted.

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Up to now most of the information on the hyperon–nucleon ( $YN$ ) interaction has been obtained either from hypernuclei or from  $K^-d$  and  $\pi^+d$  reactions. After several decades of studies our knowledge on the  $YN$  system is still far from being complete.

Recently the interest to the  $\Lambda N - \Sigma N$  system has flared again in connection to the expected CEBAF experimental results on the kaon photoproduction on the deuterium [1]. The final state  $YN$  ( $Y = \Lambda, \Sigma$ ) interaction (FSI) plays an important role in the  $\gamma d \rightarrow K^+ YN$  reaction. Therefore high resolution photoproduction experiments can substantially deepen our understanding of the  $YN$  dynamics.

The problem of FSI in  $\gamma d \rightarrow K^+ Yn$  reaction has been addressed by several authors starting from the pioneering paper by Renard and Renard [2,3,4]. Two novel features differ the present work from the previous studies. First, covariant formalism based on direct evaluation of Feynman diagrams is used which allows to analyse the data beyond the region of low energy and low momentum transfer. Second, the  $YN$  interaction is described within the  $P$ -matrix approach which takes into account the subnuclear degrees of freedom and disentangle the dynamical singularities from kinematical threshold effects [5]. The  $P$ -matrix analysis of the  $YN$  interaction was presented in [6] (see also [7]) and we shall use the set of parameters from [6].

In our previous publication[8] we have presented some preliminary results without proving the central assertion, namely that the FSI effects allow remarkably simple evaluation within the  $P$ -matrix approach. The proof of this statement along with the presentation of the covariant reaction formalism contributes the core of the present publication. Toolled with the methods presented below one can easily treat FSI within any other approach making use of the known relation between  $P$ -matrix and potential approaches [9].

The double differential cross section of the reaction  $\gamma d \rightarrow K^+ Yn$  reads

$$\frac{d^2\sigma}{d|\mathbf{p}_K|d\Omega_K} = \frac{1}{2^{11}\pi^5} \frac{\mathbf{p}_K^2}{kM_d E_K} \frac{\lambda^{1/2}(s_2, m_Y^2, m_n^2)}{s_2} \int d\Omega_{Yn}^* |T|^2. \quad (1)$$

Here  $k$ ,  $\mathbf{p}_K^2$ ,  $E_K$  and  $\Omega_K$  correspond to the deuteron rest system with  $z$ -axis defined by the incident photon beam direction  $\mathbf{k}$ . The solid angle  $\Omega_{Yn}^*$  is defined in the  $Yn$  center-of-

momentum system,  $s_2 = (p_Y + p_N)^2$ ,  $\lambda(x, y, z) = x^2 - 2(y + z)x + (y - z)^2$ .

The fully covariant analogue of (1) valid in any reference frame has the form

$$\frac{d^2\sigma}{ds_2 dt_1} = \frac{1}{2^{10}\pi^4\lambda(s, 0, M_d^2)} \int dt_2 ds_1 \frac{|T(t_1, s_1, s_2, t_2)|^2}{[-\Delta_4(t_1, s_1, s_2, t_2)]^{1/2}}, \quad (2)$$

where  $s = (k + p_d)^2$ ,  $s_1 = (p_K + p_Y)^2$ ,  $s_2 = (p_Y + p_N)^2$ ,  $t_1 = (k - p_K)^2$ ,  $t_2 = (p_d - p_n)^2$ , and  $\Delta_4$  is a  $4 \times 4$  symmetric Gram determinat [10]. The region of integration in (2) has to satisfy  $\Delta_4 \leq 0$ . The number of essential final state Lorentz scalar variables is 4; namely  $t_1, s_1, s_2, t_2$ .

The amplitude  $T$  will be approximated by the two dominant diagrams, namely the tree (pole, or plane waves) graph and the loop (triangle) diagram with  $YN$  FSI. Consider first the tree diagram. Two blocks entering into it have to be specified: (i) the elementary photoproduction amplitude  $M^{\gamma K}$  on the proton, and (ii) the deuteron vertex  $\Gamma_d$ . The elementary amplitude used in the present calculation was derived from the tree level effective Lagrangian with the account of several resonances in  $s, t$  and channels [11]. This amplitude has the following decomposition over invariant terms [12]

$$M^{\gamma K} = \bar{u}_Y \sum_{j=1}^6 \mathcal{A}_j \mathcal{M}_j(s', t', u') u_p, \quad (3)$$

where  $s' = (k + p_p)^2$ ,  $t' = (k - p_K)^2$ ,  $u' = (k - p_Y)^2$ .

The decomposition of the deuteron vertex function  $\Gamma_d$  in independent Lorentz structures has the form [13]

$$\Gamma_d = \sqrt{m_N} \left[ (p_p + p_n)^2 - M_d^2 \right] \left[ \varphi_1(t_2) \frac{(p_p - p_n)_\mu}{2m_N^2} + \varphi_2(t_2) \frac{1}{m_N} \gamma_\mu \right] \mathcal{E}^\mu(p_d, \lambda). \quad (4)$$

Here  $\mathcal{E}^\mu(p_d, \mu)$  is the polarization 4-vector of the deuteron with momentum  $p_d$  and polarization  $\lambda$ . The vertex (4) implies that the deuteron as well as the spectator neutron are on mass shell while the proton is off its mass shell. Now we can write the following expression for the tree diagram

$$T^{(t)} = \bar{u}_Y \left\{ \left( \sum_{j=1}^6 \mathcal{A}_j \mathcal{M}_j(s', t', u') \right) S(p_p) \Gamma_d \right\} u_n^c, \quad (5)$$

where  $S(p_p)$  is the proton propagator and  $u_n^c$  is a charge conjugated neutron spinor. The deuteron vertex in (5) may be substituted by the relativistic deuteron wave function according to  $\psi_d = [2(2\pi)^3 M_d]^{1/2} S(p_p) \Gamma_d$  [14]. Then (5) can be rewritten as

$$T^{(t)} = [(2\pi)^3 2M_d]^{1/2} M^{\gamma K} \psi_d, \quad (6)$$

where  $\psi_d$  is the relativistic deuteron wave functions discussed at length in [14], and where summation over magnetic quantum numbers is tacitly assumed.

The loop diagram with  $YN$  rescattering is given by the expression

$$T^{(l)} = \int \frac{d^4 p_n}{(2\pi)^4} \bar{u}_Y(p'_Y) \left\{ \left( \sum_{j=1}^6 \mathcal{A}_j \mathcal{M}_j \right) S(p_p) \Gamma_d C S(p_n) T_{YN} S(p_Y) \right\} \bar{u}(p'_N). \quad (7)$$

Here  $C$  is the charge-conjugation matrix,  $T_{YN}$  is a hyperon-nucleon vertex, this vertex being “dressed” by corresponding spinors constitutes the hyperon-nucleon amplitude  $F_{YN}$ .

The comprehensive treatment of the loop diagram will be presented in the forthcoming publication while here we resort to a simple approximation with the aim to exposure the effects of the FSI. Only positive frequency components are kept in the propagators  $S(p_n)$  and  $S(p_Y)$  in (7), while the propagator  $S(p_p)$  together with  $\Gamma_d$  is lumped into the relativistic deuteron wave function  $\psi_d$  [14]. Then integration over the time component  $dp_n^0$  is performed. Thus we arrive at the following expression for  $T^{(l)}$

$$T^{(l)} = [(2\pi)^3 2M_d]^{1/2} \int \frac{d\mathbf{p}_n}{(2\pi)^3} \frac{M^{\gamma K} \psi_d(p_n) F_{YN}(E_{YN}; q, q')}{\mathbf{q}^2 - \mathbf{q}'^2 - i0}, \quad (8)$$

where  $\mathbf{q}$  and  $\mathbf{q}'$  are the  $YN$  c.m. momenta before and after the  $YN$  FS,  $\mathbf{q} = \mathbf{p}_n - \frac{1}{2}(\mathbf{k} - \mathbf{p}_k)$ . The quality  $F_{YN}(E_{YN}; qq')$  is the half-off-shell  $YN$  amplitude at  $E_{YN} = \mathbf{q}'^2/2\mu_{YN} \neq \mathbf{q}^2/2\mu_{YN}$ . The use of the nonrelativistic propagator in (8) is legitimate since FSI is essential at low relative  $YN$  momenta. The arguments of the elementary amplitude  $M^{\gamma K}$  are specified in (3) but in the kinematical region where  $YN$  FSI is essential  $M^{\gamma K}$  can be considered as point-like and hence  $M^{\gamma k}$  can be factored out of the integral (8) at the values of the arguments fixed by the energies and momenta of the initial and final particles (i.e. at the values of the arguments corresponding to the plane-waves diagram).

Next we consider the  $YN$  rescattering amplitude  $F_{YN}$  and recall the interpretation of the  $P$ -matrix in terms of the underlying coupled-channel quark-hadron potential [15]. Namely, the  $P$ -matrix description is equivalent to the coupling between hadron and quark channels via the nonlocal energy dependent potential of the form [15]

$$V_{hgh} = \sum_n \frac{f_n(r)f_n(r')}{E - E_n}, \quad (9)$$

where  $E_n$  are the energies of the six-quark "primitives" [9,5] ( $P$ -matrix poles), and the form-factors are given by  $f_n(r) = \lambda_n \delta(r - b)$ , where  $b$  is the bag radius [9,5] and the coupling constants  $\lambda_n$  are related to the residues of the  $P$ -matrix via

$$P = P_0 + \sum_n \frac{\lambda_n^2}{E_n - E}. \quad (10)$$

As it was shown in [6] a single primitive at  $E_n = 2.34 \text{ GeV}$  is sufficient for a high quality description of the existing  $YN$  experimental data. In order to avoid lengthy equations we consider in the present note the region close to  $\Lambda N$  threshold while the preliminary results which included  $\Lambda N - \Sigma N$  coupling were announced in [8] and will be treated within the present approach in a forthcoming detailed publication.

The momentum-space half-of-shell amplitude  $F_{YN}(\mathbf{q}, \mathbf{p}; E_{YN})$  corresponding to the potential (9) reads

$$F(E_{YN}; q, q') = -\lambda_n^2 b^2 \frac{\sin qb}{qb} d^{-1}(E_{YN}) \frac{\sin q'b}{q'b}, \quad (11)$$

$$d(E_{YN}) = E_{YN} - E_n + \frac{\lambda_n^2}{q} e^{iq'b} \sin q'b. \quad (12)$$

This form of the amplitude allows to perform momentum integration in (8) analytically with an accuracy sufficient to display the effects due to FSI. In the complex  $q$  plane the integral (8) picks up contributions from the poles of the propagator and the singularities of the deuteron wave function. In a separate detailed publication we show both numerically and using models for  $\psi_d$  that the contribution of the deuteron wave function singularities does not exceed 20% (this conclusion can be immediately verified considering the simplest

singularity of  $\psi_d$  at  $q^2 = -\alpha^2$ ). Thus substituting (11-12) into (8) and performing integration with the above remark in mind we get

$$T^{(l)} \simeq [(2\pi)^3 2M_d]^{1/2} M^{\gamma K} \psi_d(p'_n) \left[ \frac{1}{f(q')} - 1 \right], \quad (13)$$

where  $p'_n$  is related to  $q'$  in the same way as  $p_n$  to  $q$  (see above), and where

$$f(q') = 1 - \frac{\lambda_n^2 b}{\Delta - q'^2/2\mu_{YN}} e^{iq'b} \frac{\sin q'b}{q'b}, \quad (14)$$

and  $\Delta = E_n - m_Y - m_N$ ,  $\mu_{YN}$  is the  $YN$  reduced mass. One can easily verify that  $f(q')$  is the Jost function corresponding to the potential (9) (some authors use the notation  $f(-q')$ ). From (14) and (6) a remarkably simple expression for the sum of the tree and loop diagrams follows

$$T^{(t)} + T^{(l)} = T^{(t)}/f(q'), \quad (15)$$

where the final state  $YN$  momentum  $q'$  is expressed in terms of the  $YN$  invariant mass  $s_2^{1/2}$  as  $q' = \frac{\lambda^{1/2}(s_2, m_Y^2, m_n^2)}{2s_2^{1/2}}$ .

Expression (14) is physically absolutely transparent: one immediately recognizes the standard enhancement factor [16] given by the inverse Jost function corresponding to the potential (9). Inclusion of the  $\Lambda N - \Sigma N$  coupling is straightforward [6]. The above equations may be used beyond the  $P$ -matrix approach to the  $YN$  interaction since there is a well trotted path connecting  $P$ -matrix and  $S$ -matrix approaches.

In conclusion we present the results of the calculation obtained using equations covariant equation (2), Eqs. (6), and (14). Use has been made of the elementary photoproduction amplitude from [11], the deuteron vertex function taken from the relativistic Gross model [14] and the plane-waves diagram with this input was calculated in [17]. In Fig.1 the double differential cross section (1) is shown as a function of the photon energy in the  $\Lambda n$  invariant mass region close to the threshold ( $2.05 GeV \leq \sqrt{s_{\Lambda n}} \leq 2.10 GeV$ ). The enhancement of the cross section close to threshold due to FSI is quite pronounced. We remind that according to [6] the  $^3S_1$   $\Lambda n$  scattering length is 1.84 fm in line with some versions of the Nijmegen potential [18].

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Fig. 1. The double differential cross section as a function of the photon energy for  $p_K = 0.861 GeV$ ,  $\theta_{\gamma K} = 0^0$ . The dashed line is the plane waves contribution, the solid line incorporates FSI according to Eq. (14).